Pre Calculus 12
Conics - Completing the Square Review

What we already know.

$$(x+3)^2 = x^2 + 3x + 3x + 9$$

We are going to work this backwards, a technique called completing the square.

Let's try one:

$$x^2 + 10x + 60$$

Notice that this equation is not a perfect square, but that doesn't matter. We are only concerned with the first two terms.

Let's rewrite this as:

$$x^2 + 10x + 25 + 35$$

Now the first three terms should look familiar. We finish this by:

$$(x+5)^2 + 35$$

Let's try another

Complete the square:

$$x^2 + 10x$$

At first this may look impossible but we can do this by adding zero to the equation.

$$x^2 + 10x + 0$$

Now this may not look like it helped, but if we find another way to represent 0, such as:

$$0 = 25 - 25$$

We can now substitute this into the equation and the solution should be more obvious.

$$x^2 + 10x + 25 - 25$$

$$(x + 5)^2 - 25$$

How do we find the number?

Complete the square:

$$x^2 - 12x + 11$$

We need to find the number that works by using the second term. To do this do the following:

$$\left(\frac{-12}{2}\right)^2 = 36$$

Now we can add and subtract this number.

$$x^2 - 12x + 36 - 36 + 11$$

Simplify to get:

$$(x-6)^2 - 25$$

A leading coefficient

Complete the square:

$$2x^2 - 14x + 9$$

First we factor out the 2 to get:

$$2(x^2 - 7x) + 9$$

For now we will be working in the brackets, complete the square. So the equation becomes:

$$2(x^2 - 7x + \frac{49}{4} - \frac{49}{4}) + 9$$

$$2\left((x - \frac{49}{4})^2 - \frac{49}{4}\right) + 9$$

Distribute the outside 2:

$$2(x - \frac{49}{4})^2 - 2(\frac{49}{4}) + 9$$

Simplify

$$2(x - \frac{49}{4})^2 - \frac{31}{2}$$

We can use this method to solve:

Solve by completing the square:

$$x^2 + 8x - 31 = 0$$

$$x^2 + 8x + 16 - 16 - 31 = 0$$

$$(x+4)^2 - 47 = 0$$

$$(x+4)^2 = 47$$

$$x + 4 = \pm\sqrt{47}$$

$$x = -4 \pm \sqrt{47}$$