Conics Pretest

Section 1 - Recognizing conics (roughly 20%)

There will be a few questions of the form:
Identify the type of conic section whose equation is given.

$x^2 = y + 1$
In this equation we do not have a y^2 term, so we know this is a **parabola**.

$x^2 = 4y - 2y^2$
First we need to rearrange this equation to get all of the terms on the left hand side.
$x^2 - 4y + 2y^2 = 0$
Now since both the x^2 and y^2 terms are positive it is an **ellipse**.

$y^2 + 2y = 4x^2 + 3$
Again, first bring all the terms to the left hand side.
$y^2 + 2y - 4x^2 - 3 = 0$
Now since exactly one of the x^2 and y^2 terms is negative it is a **hyperbola**.

$x^2 = y^2 + 1$
Rewrite as:
$x^2 - y^2 - 1 = 0$
Now since exactly one of the x^2 and y^2 terms is negative it is a **hyperbola**.

$y^2 - 8y = 6x - 18$
Here we can see that we only have a y^2 term, so we know this is a **parabola.**

$4x^2 + 6y^2 = 21$
Now since both the x^2 and y^2 terms are positive it is an **ellipse**.

$4x^2 + 6y = 21$
In this equation we do not have a y^2 term, so we know this is a **parabola**.

$4x^2 - 6y^2 + 2x - 3y = 21$
Now since exactly one of the x^2 and y^2 terms is negative it is a **hyperbola**.

Section 2 - Graphing (roughly 30%)

In this section, the equations will require minimal manipulation to put them into standard form. The questions will be of the form:

Graph the following parabola. Include the following information on your graph: vertex, axis of symmetry, focus, directrix.

$(x-3)^2 = 12y - 4$

This is almost in standard form for a parabola with a vertical axis: $(x-h)^2 = 4p(y-k)$

$(x-3)^2 = 12(y-\frac{1}{3})$ $\leftarrow$ factor out the 12

$(x-3)^2 = 12(y-\frac{1}{3})$ $\leftarrow 4p = 12 \Rightarrow p = 3$

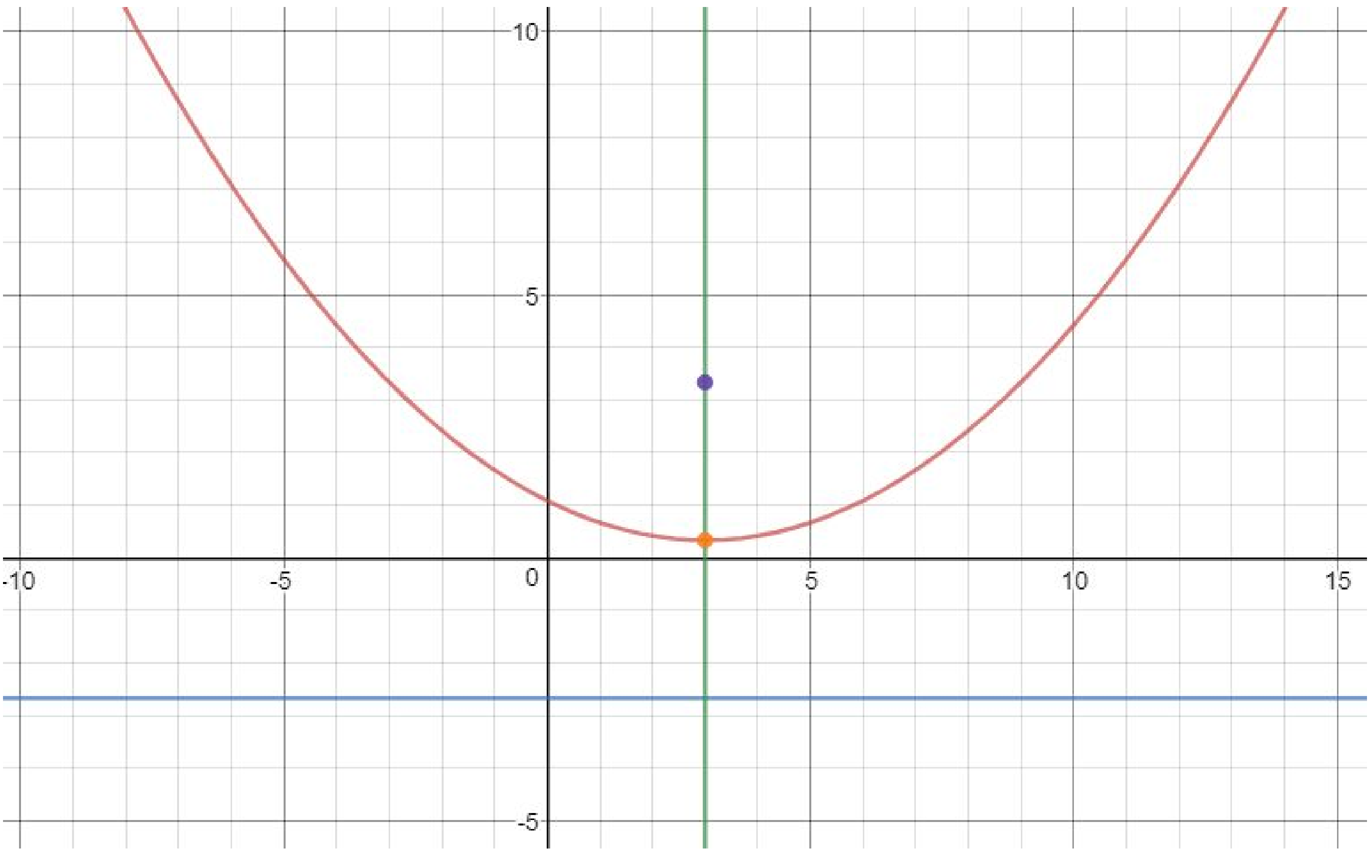
$(x-3)^2 = 4(3)(y-\frac{1}{3})$ $\leftarrow$ Substitute $12 = 4(3)$ and now it is in standard form.

Axis of symmetry: $x = h = 3$

Vertex: $(h, k) = (3, \frac{1}{3})$

Focus: $(h, k+p) = (3, \frac{1}{3}+3) = (3, \frac{10}{3})$

Directrix: $y = k - p = \frac{1}{3} - 3 = \frac{-8}{3}$



Graph the following ellipse. Include the following information on your graph: center, vertices, co-vertices, and foci.

$(x+2)^2 + 4y^2 - 36 = 0$

Since both the x^2 and y^2 terms are positive it is an **ellipse**. So we need to make the equation look like one of the following formulas. (remember for an ellipse $a > b$ determines which formula)

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

$(x+2)^2 + 4y^2 - 36 = 0$

$(x-(-2))^2 + 4(y-0)^2 - 36 = 0$ ← writing it like this so we have the h and k values

$(x-(-2))^2 + 4(y-0)^2 = 36$

$\frac{(x-(-2))^2}{36} + \frac{4(y-0)^2}{36} = 1$ ← dividing both sides of the equation by 36

$\frac{(x-(-2))^2}{36} + \frac{(y-0)^2}{9} = 1$ ← cancelling the $\frac{4}{36} = \frac{1}{9}$

$\frac{(x-(-2))^2}{6^2} + \frac{(y-0)^2}{3^2} = 1$ ← now it is in standard form of an ellipse with a horizontal major axis

Center: $(h, k) = (-2, 0)$

Vertices: $(h \pm a, k) = (-2 \pm 6, 0)$

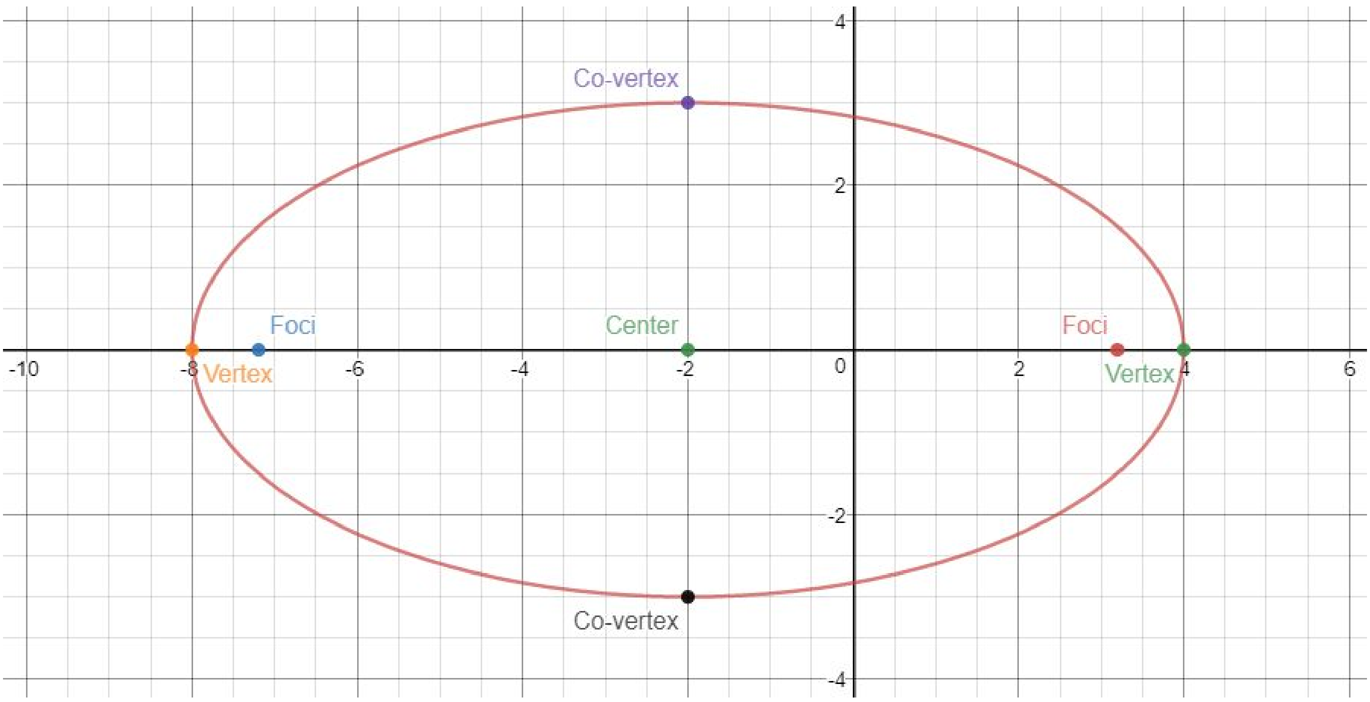
Co-vertices: $(h, k \pm b) = (-2, \pm 3)$

Foci: $(h \pm c, k) = (-2 \pm 3\sqrt{3}, 0)$

To find the foci, first we need to solve for c:

$c^2 = a^2 - b^2 = 36 - 9 = 27$

$c = \pm\sqrt{27} = \pm 3\sqrt{3}$



Graph the following hyperbola. Include the following information on your graph: center, vertices, foci and asymptotes.

$(y+1)^2 - 4(x-1)^2 = 100$

We need to make this look like one of the following:

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ or $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$(y+1)^2 - 4(x-1)^2 = 100$

$(y-(-1))^2 - 4(x-1)^2 = 100$ ← writing it like this so we have the h and k values

$\frac{(y-(-1))^2}{100} - \frac{4(x-1)^2}{100} = 1$ ← divide both sides by 100

$\frac{(y-(-1))^2}{100} - \frac{(x-1)^2}{25} = 1$ ← simplify the $\frac{4}{100} = \frac{1}{25}$

$\frac{(y-(-1))^2}{10^2} - \frac{(x-1)^2}{5^2} = 1$ ← Now in standard form

Center: $(h, k) = (1, -1)$

Vertices: $(h,\ k \pm a) = (1, -1 \pm 10)$

Foci: $(h, k \pm c)\ = (1,\ -1 \pm 5\sqrt{5})$

To find the foci we need to solve:

$c^2 = a^2 + b^2 = 100 + 25$

$c = \ \pm\sqrt{125} = \ \pm 5\sqrt{5}$

Asymptote equation:

$y = k \pm \frac{a}{b}(x-h)$

$y = \ -1 \pm \frac{10}{5}(x-1)$

$y = \ -1 \pm 2(x-1)$

$y = \ -1 \pm 2(x-1)$

Positive:

$y = \ -1 + 2(x-1)$

$y = \ -1 + 2x - 2$

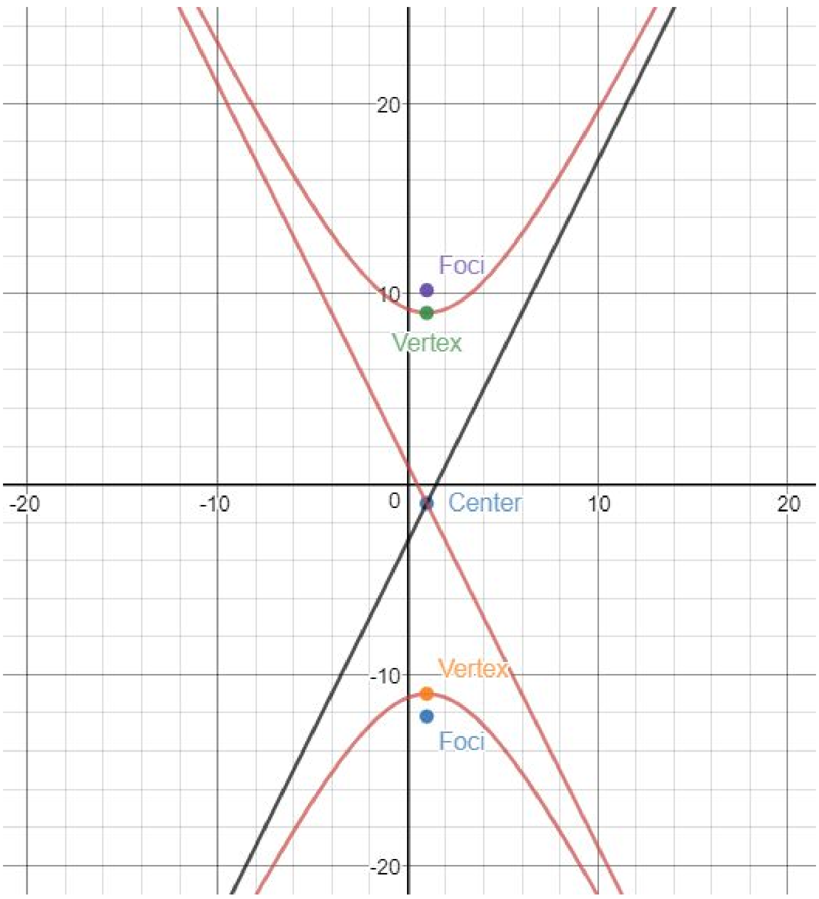
$y = \ 2x - 3$

Negative:

$y = \ -1 - 2(x-1)$

$y = \ -1 - 2x + 2$

$y = \ -2x + 1$



Section 3 - Manipulating Equations (roughly 30%)

In this section, the equations will require more manipulation than in the previous section (think completing the square). The questions will be of the form:

Identify the conic and rewrite the equation in standard form:

$x^2 - 4x - 2y + 18 = 0$

$(x^2 - 4x + \quad - \quad) - 2y + 18 = 0$ ← we need to add and subtract $(\frac{4}{2})^2$

$(x^2 - 4x + 4 - 4) - 2y + 18 = 0$

$((x - 2)^2 - 4) - 2y + 18 = 0$ ← $(x^2 - 4x + 4 - 4) = ((x - 2)^2 - 4)$

$(x - 2)^2 - 4 - 2y + 18 = 0$

$(x - 2)^2 - 2y + 14 = 0$

$(x - 2)^2 = 2y - 14$

$(x - 2)^2 = 2(y - 7)$

Now we need to solve: $4p = 2 \Rightarrow p = \frac{1}{2}$

$(x - 2)^2 = 4p(y - 7)$

$(x - 2)^2 = 4(\frac{1}{2})(y - 7)$

$4x^2 + 32x + 25y^2 - 150y - 111 = 0$

$4(x^2 + 8x) + 25(y^2 - 6y) - 111 = 0$ ← factoring out the terms in front of the squares

$4(x^2 + 8x + 16 - 16) + 25(y^2 - 6y + 9 - 9) - 111 = 0$ ← starting to complete the square

$4((x + 4)^2 - 16) + 25((y - 3)^2 - 9) - 261 = 0$ ← completing the square

$4(x + 4)^2 - 4(16) + 25(y - 3)^2 - 25(9) - 111 = 0$ ← distributing the 4 and the 9

$4(x + 4)^2 - 64 + 25(y - 3)^2 - 225 - 111 = 0$

$4(x + 4)^2 + 25(y - 3)^2 = 400$

$\frac{4(x+4)^2}{400} + \frac{25(y-3)^2}{400} = 1$ ← simplifying the fractions

$\frac{(x+4)^2}{100} + \frac{(y-3)^2}{16} = 1$

$\frac{(x+4)^2}{10^2} + \frac{(y-3)^2}{4^2} = 1$

$2x^2 - 4x - 8y^2 - 16y - 14 = 0$

$2(x^2 - 2x) - 8(y^2 + 2y) - 14 = 0$

$2(x^2 - 2x + 1 - 1) - 8(y^2 + 2y + 1 - 1) - 14 = 0$

$2((x - 1)^2 - 1) - 8((y + 1)^2 - 1) - 14 = 0$

$2(x - 1)^2 - 2 - 8(y + 1)^2 + 8 - 14 = 0$

$2(x - 1)^2 - 8(y + 1)^2 = 8$

$\frac{(x-1)^2}{4} - \frac{(y+1)^2}{1} = 1$

$\frac{(x-1)^2}{4} - \frac{(y-(-1))^2}{1} = 1$

Section 4 - Understanding the properties of conics (roughly 20%)

In this section, you will be given some information about a conic and be asked to determine further information (beyond just using formulas). These questions will be similar to the following:

1. **Find the vertex of a parabola with the directrix x=2 and focus at (6,-1).**
   Hint: the vertex is halfway between the directrix and the focus.

2. **The point (7,3) lies on a parabola with the axis of symmetry y=1. Find another point on the parabola.**
   Hint: think about what symmetry means

3. **A circle has vertices of (-4,8) and (-12,8), find the other two vertices.**
   Hint: find the center of the circle and the radius

4. **A hyperbola has vertices at** $y = 1 \pm \frac{3}{4}(x-2)$**. Find the center.**
   Hint:sometimes the formula is easier to use then solving equations